Differential heterogenesis and the emergence of semiotic function

Alessandro Sarti, Giovanna Citti, David Piotrowski

Differential heterogenesis and the emergence of semiotic function

Alessandro Sarti [*], Giovanna Citti [†], David Piotrowski [‡]

May 31, 2018

In this study, we analyse the notion of "differential heterogenesis" proposed by Deleuze and Guattari on a morphogenetic perspective. We propose a mathematical framework to envisage the emergence of singular forms from the assemblages of heterogeneous operators. In opposition to the kind of differential calculus that is usually adopted in mathematical-physical modelling, which tends to assume a homogeneous differential equation applied to an entire homogeneous region, heterogenesis allows differential constraints of qualitatively different kinds in different points of space and time. These constraints can then change in time, opening the possibility for new kinds of differential dynamics and the emergence of distinct entities and forms. Formally, we show that operators with different phase spaces can be assembled on the basis of a result of Rotschild and Stein (L. Rothschild, E. M. Stein , 1976). Furthermore, operators with different dynamics can be assembled by means of a partition of the unit.

After stating the concept of differential heterogenesis in terms of contemporary mathematics, we show that this construction sheds light on the constitution of the semiotic function. In fact, both the Merleau-Pontian and the Deleuzian approaches share a common conceptualisation of the semiotic function and its emergence in terms of a *morphodynamics of heterogeneous assemblages with a divergent actualisation*. This divergent actualisation allows the co-constitution of various expression and content planes. Finally, we show that the divergent actualisation can be interpreted as the directions of principal eigenvectors of the actualized flow.

[*]Alessandro Sarti, Center for Mathematics, EHESS/CNRS, Paris
[†]Giovanna Citti, Department of Mathematics, University of Bologna
[‡]David Piotrowski, LIAS, EHESS/CNRS, Paris

1 Introduction

Semiosis is a generative and emergent process as clearly stated by Paolo Fabbri (Fabbri , 1998), who states that "differently from logic, the point is not to construct preliminary logical systems and then to see how they work in language, but to assemble some very simple units and then observe emergent properties" (ibid., our translation). The aim is to "see how, at a certain point, some emergent properties organize themselves and acquire meaning, that is exactly the opposite of old semiotics and old logics" (ibid., our translation).

From the point of view of structural semiotics, signification processes presuppose systems of units with identities that are differential and positional. Dynamical structuralism developed in the main works of Réné Thom (R. Thom , 1972) and Jean Petitot (J. Petitot , 2017), who have deeply studied systems of oppositions engendered by qualitative discontinuities in morphodynamics, proposing to model several semiotic systems with catastrophe theory.

But the ontogenetical issue has to be faced now on new bases that take into account the globality of the morphogenetical process, including the emergence of the semiotic function and of the semiotic spaces in which systems of oppositions will be installed. Far before stabilization of dynamics in the basins of attraction, there is a progressive individuation of the space itself in which the landscape of potentials lives.

Far before that categorisation is performed by sign (D. Piotrowski , 2017), a multitude of symptomatic elements contribute to constructing a protosemiotic space already equipped by its semiotic function without the presence of any stabilization.

This perspective has been recently reconsidered in "Individuation and Morphogenesis" (A. Sarti, F. Montanari, F. Galofaro eds. , 2015) from the point of view of the concept of *individuation* introduced by Gilbert Simondon in his thesis (G. Simondon , 1964). With the idea of individuation, Simondon interprets the becoming of forms as a continuous passage from a pre-individual intensive plane to an emergent plane of extensive forms. He considers this process at different scales and levels of complexity: physical, biological, psychological and trans-individual.

Deleuze (G. Deleuze , 1994) clarifies that the individuation process has a differential origin and that the becoming of forms is defined as the solution of a differential problem. Following Deleuze, the differential becoming is a passage from the virtual to the actual, in which the virtual is a distribution of differential operators and the actual is the solution to the corresponding differential problem. This distribution of operators is heterogeneous, since the differential operators are all different one to the other. For this reason, they are called "singular", in the sense of unique and specific. This singular

distribution is intensive and cannot be perceived, since it does not belong to the phenomenal plane. The integration of the differential constraints gives rise to forms, perceptions and extensive morphologies in the endless becoming of the differential heterogenesis.

In this study, we propose a mathematical formulation of heterogenetic becoming. We analyse how progressive polarisation of the heterogenetic flow allows the emergence of the expression/content stratification.

Rather than a philological analysis of the Deleuzian transcendental empiricism, which we leave to specialists (see, for example, Anne Sauvagnargues (A. Sauvagnargues , 2008)), we prefer to draw inspiration from its guiding ideas to envisage the possibility of constructing a mathematics of heterogeneous becoming. Some recent contributions on the role of mathematics in the philosophy of Gilles Deleuze have been sources of inspiration and for the development of the present study, such as (A. Longo , 2016) (I. Krtolica , 2015) (T. May , 2005).

In the first part of the paper, we will outline that, unlike differential processes in mathematical physics, in which differential operators are invariant in a certain phase space and they are given as invariant laws, in heterogenesis, the differential constraints are singular and are composed to build different assemblages that are always different (agencement). The operation of composition of assemblages is similar to an action of composition of differentials that are modified, added, eliminated and in general recombined in new configurations. This action corresponds to a true plastic assembly of a multiplicity of differentials on the virtual plane.

Assemblages define time by time their own spaces. In this case, the morphogenetical space is not given *a priori* as in mathematical physics, but it is a consequence of assemblage of singularities. Together with a morphogenesis *in* the space, we have also a morphogenesis *of* the space, since assemblages are continuously evolving.

In the second part of the study, we will introduce the emergence of the semiotic function as progressive polarization of the heterogenetic flow leading to the separation of E/C planes, each one containing its own formed substances.

While the dynamics of separation of the two planes are largely unknown and typical to each semiotic context, we will speculate on the possibility of a spectral differentiation of the planes, in which the eigenvectors of the assemblage will indicate the independent directions of subspaces E/C. This construction allows the emergence of the semiotic function from the dynamic evolution of the heterogenetic flow without the need of any stabilisation, as opposed to the classical case of structural morphodynamics.

Before arriving at this conclusion and to prepare the reader for it, we discuss the main approaches to the construction of the semiotic function that have been proposed in classic literature (Hjelmslev, Husserl, Peirce, Eco, Fontanille,

and Merleau-Ponty).

2 Elements of Heterogenesis

In Difference and Repetition (G. Deleuze , 1994) Gilles Deleuze proposes a concept of becoming that is largely based on the Simondonian idea of individuation. Individuation means passage from a pre-individual field to an individuated one. Deleuze specifies this passage as from a virtual plane to an actual one or in terms of a transformation from the virtual plane to its actualization.

Unlike Simondon, Deleuze characterizes this passage in a specific mathematical way. He reconsiders the concepts of differential of Leibniz and defines the virtual as a multiplicity of differential operators.

This distribution is intensive and not perceivable.

The perceived forms, as well as the mental forms of thinking and more generally any morphogenetical process, are nothing but the solution of a problem posed by the multiplicity of differential constraints that constitute the virtual. In other words, the origin of any morphogenesis is differential.

Even if differential calculus is just a mathematical tool, the differential becoming is considered a general dialectic that overcomes mathematics: "il trouve son sens dans la révélation d'une dialectique qui dépasse la mathématique"(F. de Saussure , 1959). Problems that are mathematical, physical, biological, sociological, and semiotic find their solutions in different disciplines by actualizing differentials in a proper manner. In any case, the solutions emerge always by integration of "un système de liaisons entre éléments différentiels, un système de rapports différentiels entre éléments génétiques. Si l'Idée est la différentielle de la pensée, il y a un calcul différentiel correspondant à chaque Idée, alphabet de ce que signifie penser" (A. Sarti, F. Montanari, F. Galofaro eds. , 2015).

The classical example of such a differential problem is the construction of a curve, which is set up by the integration of a family of tangent lines, each one carrying its proper differential constraint. In the curve integration problem, the differential constraints are homogeneous, or more precisely, equirregular, but nothing prevents leaving more freedom to differential operators and considering heterogeneous constraints.

This case is only sketched in Difference and Repetition, and it is developed together with Felix Guattari in Thousand Plateaus (G. Deleuze, F. Guattari , 1987). The concept of heterogeneous assemblage (agencement) of planes is introduced in Thousand Plateaus at a philosophical level more than at the mathematical one, but it is clearly the extension of the idea of differential becoming to a heterogeneous setting.

We freely interpret this heterogeneity at least from two different perspectives.

We find a first level of heterogeneity in the constitutive difference of differential constraints, which can induce a variety of dynamical behaviour changing point to point. A second level of heterogeneity is present since each differential constraint has its own structure of tangent planes constituting the phase space, which are the "plateaux" that fluxes are allowed to flow on. The continuously changing geometry of directions of the flows is then a further element of heterogeneity.

This heterogeneous differential problem is posed in terms of a composition of the differential constraints to form assemblages. Heterogeneous assemblages are not built on the basis of a logic compatibility or compliance, but by the possibility of differential constraints to create new spaces and new dynamics not given *a priori*, in such a way that phase spaces as well as dynamics are invented by the intrinsic construction of the singular composition.

How this heterogeneous composition is possible is one of the mathematical problems we examine in this study. How the conjunction of heterogeneous differentials is able to give rise to an agencement is a difficult mathematical problem that we will consider in the next section, starting from the work of Rothschild and Stein (L. Rothschild, E. M. Stein , 1976).

Just to envisage what a similar approach can carry on, let us consider the organization of brain dynamics. The brain is made up of neural populations with heterogeneous dynamics that are mathematically described by heterogeneous operators. At the same time, populations act on a set of neurochemicals such as neurotransmitters, messengers and neuromodulators that give rise to a heterogeneity of formed substances.

Again, neural connectivity that defines the structure of tangent planes of dynamics is different population by population. These populations are concatenated together in the form of agencement, at which point they must be considered a material implementation of heterogenesis. Finally, neural connectivity is plastically modified by learning processes that implement a true plasticity of the virtual, which corresponds to a continuous reorganization of the differential rules underlying dynamics.

Brain heterogenesis therefore constitutes the material differential layer of every phenomenology of perception and imagination, the forms of which are deployed as a solution to the differential problem (notice that Deleuze and Guattari analyse this topic in their last work (G. Deleuze, F. Guattari , 1994)).

We are quite far from the usual differential calculus of mathematical physics in which the distributions of operators are spatially and temporally homogeneous. In heterogenesis, there is a spatially and temporally varying definition of differential constraints. Mathematical physics is a form of symmetrization of heterogenesis in the sense that any heterogeneous set is reduced to a unique operator that holds in every spatio-temporal point. Heterogenesis can be re-

garded as a *Hyperphysics* that takes place as a variety of dynamics flowing on a multiplicity of tangent planes that change from point to point.

This character of "homogeneisation" of mathematical physics is at the basis of its fundamental *a priori*, presupposing that spaces are given as an *a priori* with respect to differential constraints. This *a priori* is completely reversed in the composition of heterogenetic assemblages, in which operators are primary and define dimensions and qualities of the space: a new differential singularity that is composed with an assemblage that redefines completely the spaces of the entire assemblage.

In mathematical physics, operatorial homogeneity and the fixity of the differential constraints determine the universality of laws and the nomological character of differential models.

Heterogenetic composition is poles apart from universal laws and lays the conditions for an immanent morphogenesis that is created time by time by the assembly of singular concatenations.

Notice that if the assemblage of operators is considered in turn a new differential operator, heterogenesis can be viewed as a morphogenesis of the assemblage operator. The heterogenetical becoming is then considered a concurrent morphogenesis of operators, of its spaces and of forms in spaces, a concept that is unprecedented in physical and structural dynamics.

To allow the construction of assemblages, two temporal scales or axes are present. The first is the axis of the actualisation of differential constraints. It is the axis of Kronos, which is common to mathematical physics. The second is the axis that Deleuze calls Aion, on which it takes the place the recombination of differential constraints in new assemblages. On this axis, we have a true plasticity of the virtual, meaning the possibility to recombine genetic elements to create singular dynamics. Any specific recombination has to be thought of as an explorative action, closer to a Dada performance rather than to a finalised process. It is not subjected to any mathematization. The composition of a singular assemblage is then an invention, the creation of new dynamics instant by instant. The inventive character of the assemblage is due to the fact that the space created by the assemblage is much more than the union of identitary spaces of single operators. As we will clarify in the mathematical presentation, this is because second order differences (differences of differences) increase the dimension of the tangent space and open to new planes, which was inconceivable before.

This feature well interprets the Deleuzian idea that rather than to search the common in the difference (in a process of homogeneisation of existing spaces), it is to think in a differential way about the difference. Precisely because of these differences of differences (that occur through the mathematical operation of commutation, as we will see in the next chapter), new spaces arise with all

their possible dynamics.

What is the meaning of reconsidering heterogenesis from the mathematical point of view? The first motivation relies on the fact that the very origin of Deleuzian heterogenesis has an operational nature, since Deleuze takes the Leibnizian differential calculus as a model and more generally the operational disposition of baroque culture. Differential calculus is at the basis of the idea of becoming in Difference and Repetition. Becoming assumes from the beginning a problematic dimension, in the strict mathematical sense of posing and solving a problem.

Deleuze explicitly explains the role of mathematics in its constructivist empiricism: " ... how can something be given to a subject, and how can the subject give something to itself? Here, the critical requirement is that of a constructivist logic which finds its model in mathematics. The critique is empirical when, having situated ourselves in a purely immanent point of view, which makes possible a description whose rule is found in determinable hypotheses and whose model is found in physics, we ask: how is the subject constituted in the given? The construction of the given makes room for the constitution of the subject. The given is no longer given to a subject; rather, the subject constitutes itself in the given" (G. Deleuze , 2001).

Becoming is viewed as the creative principle arising from the position of a problem in terms of a constellation of differential operators heterogeneous among themselves. This phase of plastic composition of differentials puts in place the problematic and intensive dimension of becoming, which can be regarded as a form of plasticity of the virtual. Mathematics can then be used as a language to evoke the dynamical becoming of a complex materiality endowed by its substantial consistency as a vital, singular, semiogenetic flow. For more about this idea of vital materialism, see also Rosi Braidotti (R. Braidotti , 2002) and Jane Bennet (J. Bennett , 2010).

Besides this intrinsic motivation, there is also a historical contingent factor that pushes us to elaborate mathematically heterogenesis. As in Albert Lautman's epistemic view, mathematics is considered a language that is always relative to specific and situated problematic circumstances, in which an important part of mathematical invention consists of the formulation of problems. The history of mathematics is considered a history of problems, more than an automatic progress independent from the cultural and historical context, as in the axiomatic perspective. The work of mathematicians is to envision the entire problematic dimension in an original way.

We are thus interested in the question of heterogenesis more to problematize than to offer solutions. In particular, we are interested in problematizing the question of contemporary models in life sciences and human sciences. Models in life sciences and human sciences, from the cognitive to the social point of

view, from the aesthetic to the semiotic aspect, come from a culture of physical science that considers an invariant and homogeneous distribution of operators.

This nomological use of operators is at the base of contemporary modelling culture: the Navier-Stokes equation for viscous fluids is the same in all points of space and time. Analogously, Alan Turing's (A. Turing , 1952-1992) equation of morphogenesis, deeply studied also by Réné Thom, presents spatial and temporal symmetries.

Within the realm of life sciences, a deep problematization of invariances and symmetries and the necessity of evolving phase spaces has been proposed by Giuseppe Longo (F. Bailly, G. Longo , 2008) (G. Longo, M. Montevil , 2014).

In the same way, models of mathematical and computational economics are based on the interaction of individuals endowed by the same space of rationality. These approaches are founded more or less explicitly on the paradigm of methodological individualism (A. Laurent , 1994) (J. Petitot , 2015), in which every process of individuation is reduced to a functional interaction between already individuated homogeneous units.

If homogeneous constraints well describe a form of swarm intelligence or crowd behaviour, it reduces dynamics to automatisms by excluding any imaginative and creative aspect. We aim to problematize the procedure of homogenization dominant in life and social science and to outline the dynamical heterogeneity of life and its affective, semiotic, social, and historical aspects.

The purpose is to free up the dynamic becoming from any form of unitary and totalizing symmetry and to develop forms, action, and thought by means of dispositives of proliferation, juxtaposition, and disjunction.

3 Heterogenesis as a mathematical differential problem

In classical mathematical setting (D. Gilbarg, N. S. Trudinger , 1998) (A. Friedman, , 1964) and particularly in mathematical physics, a differential problem is assigned by defining an operator on a domain Ω and suitable initial and boundary conditions.

The space is then considered uniform, and differential operators are represented in terms of a gradient ∇, which is the list of all partial derivatives $(\partial_1, \cdots, \partial_n)$, and their iteration of any order ∇^k. The expression of the operator A is the same at every point of the domain Ω: precisely

$$A(u)(p) = A(p, u(p), \nabla u(p), \nabla^2 u(p), \cdots, \nabla^k u(p)), \text{ at every point } p \text{ in } \Omega$$

To ensure the development of the theory, it is generally required that the operators satisfy suitable conditions uniformly on the whole set (indeed, the

operators are classified as uniformly elliptic, uniformly parabolic, and uniformly hyperbolic). If the condition of uniformity is removed, the operators are called degenerate, and classical theory can no longer provide existence or regularity results.

A different point of view has been recently considered by Hörmander in (L. Hörmander , 1967) and Rothshild and Stein in (L. Rothschild, E. M. Stein , 1976). They introduced a class of operators that are degenerate, since they are defined on a differential structure that can have different behaviour from one point to an other. In particular, while inverting one such operators, Rothshild and Stein defined a family of approximate operators and proved that their hull univocally defines a new operator A. In their work, all the approximating operators have the same formal expression even if the differential structure can change from one point to the other. A large literature has been originated by their work. See for example (G. Citti , 1996), (G. Citti, E. Lanconelli, A. Montanari , 2002), (G. Citti, M, Manfredini , 2005), and the review paper (L. Capogna, G. Citti , 2016).

Here we introduce a multiplicity of operators $(A_{p_i})_{i=1,2,...}$ that are different from a point to the other by removing the assumption that all the operators have the same formal expression. We also weaken some requirements of the differential structure. This multiplicity of differential operators mathematically expresses the notion of heterogeneity of the "virtual plane", introduced by Deleuze-Guattari (section 3.1). Then we formalize the passage from this virtual plane to the actual one in three steps. We first show that the differential operators induce a space not given *a priori* via a process called lifting (section 3.2). Then we will formalize a mathematical assemblage operator that is a formal re-interpretation of the Deleuzian concept of agencement (section 3.3).

Finally, the actualization of the process is accomplished as the solution of an equation associated to the assemblage operator, which is a flow, with an heterogeneous dynamic (section 3.4). In Section 3.5, we will show that the heterogenetic flow can be represented in a low dimensional system of reference created by himself, the so-called embedding. The axes of this reference system constitute the directions of cohesion in which the flow can be polarized.

3.1 Properties of each operator of the multiplicity

Let us describe a general operator A_{p_0} in the considered multiplicity of heterogeneous operators.

Here we consider two levels of heterogeneity. The first is that the heterogeneous operators are locally defined. Each will act only on functions u defined in a neighbourhood of a point p_0 that will be denoted B_{p_0}. The formal definition of the operator will not be the same at every point, but will explicitly depend

on the point p_0 and will be denoted A_{p_0} (see fig. 1).

The second level of heterogeneity is the fact that we will replace partial derivatives and their gradient ∇ with local and directional derivatives ∇_{p_0} to allow a more general differential constraint. As a consequence the expression of a local operator becomes:

$$A_{p_0}(u)(p) = A_{p_0}(p, u(p), \nabla_{p_0} u(p), \nabla^2_{p_0} u(p), \cdots, \nabla^k_{p_0} u(p)),$$

where the operator acts on functions u defined in the neighbourhood B_{p_0} of the point p_0.

Directions of the propagation induced by the operator A_{p_0} strongly depend on ∇_{p_0} (J.-M. Bony , 1969), since the higher order derivatives are obtained by applying in sequence first order derivatives. We assume that the set of allowed directions of propagation can change from on point to another, even within a neighbourhood of a fixed point p_0. Precisely at every point p_0, the operator will induce propagation along the directions of a family of vector fields that will be denoted $(\vec{v}_{1,p_0}, \vec{v}_{2,p_0}, \cdots)$, since they change from one point to the other. The associated directional derivatives that allow propagation in these directions will be denoted $(\partial_{v_1,p_0}, \partial_{v_2,p_0}, \cdots)$, and the complete list of these directional derivatives defines a non-standard gradient, $\nabla_{p_0} = (\partial_{v_1,p_0} \partial_{v_2,p_0}, \cdots)$.

At every point, we define an admissible tangent space $T_{p_0}(p)$ generated by the derivative $(\partial_{v_1,p_0}, \partial_{v_2,p_0}, \cdots)$ at any point p of the neighbourhood of p_0, which hence will have dimension and structure that are different from one point to the other.

A possible example of such heterogeneity is the organization of brain dynamics. This is made up of neural populations that are mutually heterogeneous and can be mathematically described by heterogeneous operators. Neural connectivity that defines the structure of tangent planes of dynamics is different population by population. (This situation is depicted in fig 1, in which a single neighbourhood B_{p_0} is represented and tangent spaces $T_{p_0}(p)$ of different dimension are represented in different points p.)

3.2 Geometric heterogeneity and lifting

3.2.1 Classical definitions of lifting

In classical mechanics, a lifting process defines the phase space associating to each point, its co-tangent space. For example, to every point of a trajectory in a 3D space, it is possible to associate a vector, called momentum, that corresponds to the velocity of the point. The resulting structure, the elements of which are the couples of position and momentum, has dimension 6 at every point. Two points with the same position but different velocities are different

coincides in world space, but are different in the phase space, allowing a better understanding their motion.

Other examples of lifting have been proposed by Hoffmann (W.C. Hoffman , 1989), Petitot (J. Petitot, Y. Tondut , 1999), Citti-Sarti (G. Citti, A. Sarti , 2006), Sarti-Citti-Petitot (A. Sarti, G. Citti, J. Petitot , 2008), and Duits (R. Duits, E.M. Franken , 2010) to study connectivity of a neural population. Cells in visual areas are characterized by their ability to select differential features, such as direction of boundaries, curvature, and colour, of the visual stimulus. Within a single population, behaviour is homogeneous, and each cell is able to select different instances of the same feature. Over every point p of the visual plane, a whole fibre of cells is present that is sensible of different values q of the specified feature, and the lifting associates to every point p a couple $\tilde{p} = (p, q)$ of position and feature (see fig. 1).

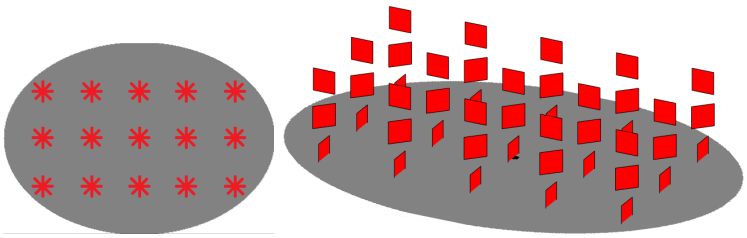

Figure 1: Left: the tangent plane has the same dimension at every point p. Right: the lifting in a higher dimensional space of position and feature (orientation in this case).

3.2.2 Lifting of heterogeneous vector fields

We can now consider an operator A_{p_0} with an heterogeneous vector field structure. Then we operate a feature selection that will lift the heterogeneous structure to a locally homogeneous space in which composition of differential operators is possible.

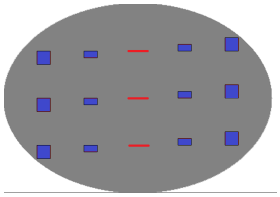

Figure 2: Visualization of the tangent planes $T_{p_0}(p)$. at all points p in a neighbourhood of the point p_0: they differ from one point to another.

As Bony (J.-M. Bony , 1969) proved, the flow associated to the operator A_{p_0} will propagate not only along the directions of admissible vector fields ∂_{v_i,p_0}, but also along the direction of new vector fields, called commutators, that can be formally expressed as differences of second order derivatives.

Indeed, if ∂_{v_1,p_0} and ∂_{v_2,p_0} are directional derivation operators, also $\partial_{v_1,p_0}\partial_{v_2,p_0} - \partial_{v_2,p_0}\partial_{v_1}$ is a directional derivative, called commutator and denoted $[\partial_{v_1,p_0}, \partial_{v_2,p_0}]$.

The algebra $L_{p_0}(p)$ contains all the admissible directional derivatives ∂_{i,p_0} and their commutators and will allow a complete description of the direction of propagation.

Hence, the algebra contains the tangent space at every point, but the two sets (vector fields and algebra) can be different. The properties of the algebra can be used to lift to a higher dimensional space, in which tangent space has the same dimension at every point.

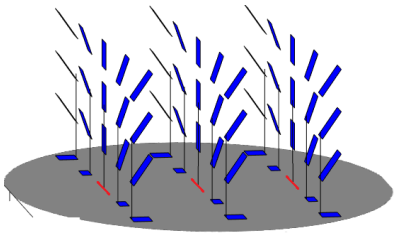

Figure 3: Visualization of the lifting: the heterogeneous tangent spaces $T_{p_0}(p)$ defined by the multiplicity of operators are lifted to tangent planes $\tilde{T}_{p_0}(p)$ which have all the same dimension, in a higher dimensional structure. The 2D projections of the lifted tangent planes $\tilde{T}_{p_0}(p)$ provide the heterogeneous ones $T_{p_0}(p)$.

To achieve this differential property, a delicate construction is applied by adding new variables. It is not sufficient to perform a Cartesian product, but some identifications and quotients on the spaces ar needed.

Each point p of B_{p_0} is lifted to a point $\tilde{p} = (p, q)$, and the domain B_{p_0} is lifted to a higher dimensional domain $\tilde{B}_{\tilde{p}_0} = (B_{p_0}, F_{p_0})$ in which tangent space and its algebra have the same dimension.

The gradient in the lifted space will be denoted $\tilde{\nabla}_{\tilde{p}_0}$, and the operators A_{p_0} at every point will be lifted to operators $\tilde{A}_{\tilde{p}_0}$. Fig. 3 visualizes the lifted space and the new family of tangent spaces $\tilde{T}_{\tilde{p}_0}$ at every point. The lifted space is homogeneous, with tangent planes of the same dimension at every point.

In this process, operators are primary and define dimensions and qualities of the space. This leads to the definition of the domain of the solution starting from the multiplicity of the differential operators. In this sense, the space is not

give *a priori*, but it is induced by the differential relation between the operators of the multiplicity.

We can also assume that gradients ∇_{p_0}, which describe the direction of propagation, are not *a priori* fixed. They depend on the dynamic evolution of the solution u. This implies that the vector fields are constraining the solution and at the same time depend on the solution. The structure of tangent planes will be different if the solution has different values. Equations of this type can present shocks and crack formation. A crack and a fracture of an object is a sudden episode, non-reproducible in the same way.

3.3 The assemblage operator

In the previous section, we have defined a multiplicity of operators A_{p_i}, and shown how to lift heterogeneous vector fields. Now we will show how this can be used as the first step of the assemblage. We will see how to construct an assemblage of this multiplicity performed by the operator A, such that $Au(p_i) = A_{p_i}u(p_i)$ for every point p_i of the set P.

This is done in two steps. First, we apply the lifting to the union of all considered operators of the multiplicity. Then we construct the assemblage operator in the lifted space.

3.3.1 Lifting a multiplicity of patches

The differential operators and the lifted structure are well defined only in a small neighbourhood of each point p_0. They are not globally defined.

If we assign two points, p_0 and p_1, they can be connected only if the associated neighbourhoods B_{p_0} and B_{p_1} have a non-vanishing intersection.

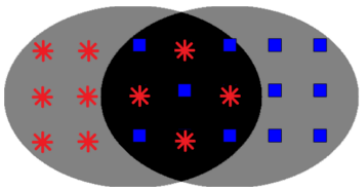

Figure 4: Intersection of homogeneous regions are necessarily heterogeneous.

Even if we assume that within each patch the vector field is homogeneous, in the intersection of patches, the vector fields are necessary heterogeneous, since we can collect all the directional derivatives of the two different gradients in a new gradient $\nabla_{p_0,p_1} = (\nabla_{p_0}, \nabla_{p_1})$ defined in the intersection $B_{p_0} \cap B_{p_1}$ of the two patches (see fig. 4).

Hence, we have here to apply both the homogeneous and heterogeneous lifting defined in the previous section. The domains will be lifted to new domains $\tilde{B}_{p_i} = (B_{p_i}, F_{p_i})$, vector fields will be lifted to vector fields $\tilde{\nabla}_{p_i}$ that coincide in the intersection of patches with the lifting of the operator ∇_{p_0,p_1}, and the operators will be lifted to new operators $\tilde{A}_{p_i}$.

The heterogeneous assemblages are not unique and are defined only on the basis on the possibility of differential constraints to create new spaces and new operators. Indeed, we could either first intersect the operators (as in fig. 4) and then perform the lifting, or, alternatively, lift the two neighbourhoods separately to intersect the patches and lift again (see fig. 5).

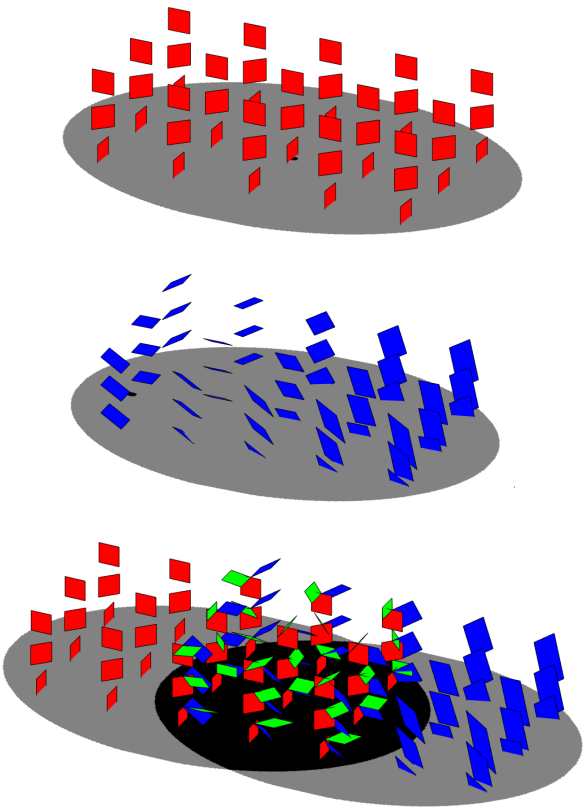

Figure 5: Lifting of the assemblage. Top: Lifting of the operator A_{p_0}. Middle: Lifting of the operator A_{p_1}. Bottom: Lifting of the assemblage $\&A_{p_0,p_1}$. Notice that the lifting of the assemblage is performed by the generators induced by A_{p_0}, the generators induced by A_{p_1} and their commutators (in green). Then the lifting of the assemblage is more than the union of the separated liftings, due to the presence of new commutators (differences of differences in the language of Gilles Deleuze). This assembly of planes indicates the possible directions of flows.

.

The lifting $\tilde{\nabla}_{p_0,p_1}$ contains commutators that did not exist in each of the lifted operators separately, and the interaction is much more than the simple union of the collected vector fields. Commutators interpret in a formal way the differences of differences, which are so important in the Deleuzian construction of assemblages.

3.3.2 Partition of the unit and assemblage

The two operators $\tilde{A}_{p_0}$, and $\tilde{A}_{p_1}$ are defined on different neighbourhoods, which makes it difficult to define a deformation of one operator on the other to formalize the notion of assemblage. However, we can extend both of them to the whole space assigning value 0 outside its domain $\tilde{B}_{p_i} = (B_{p_i}, F_{p_i}$. This can be made by multiplying each of them by a function ϕ_i that has value 1 around the point p_i and 0 outside the set $\tilde{B}_{p_i}$. If we also normalize the two functions in such a way that their sum is identically 1, the couple (ϕ_1, ϕ_2) is called partition of unit.

Thanks to this extension, the two operators act on the same set of functions, allowing heterogeneous assemblage, which is a deformation of an operator in the other. This process of assemblage can be formally expressed as a linear combination of the two lifted operators:

$$\&\tilde{A}_{p_0,p_1} = \phi_0 \tilde{A}_{p_0} + \phi_1 \tilde{A}_{p_1}$$

Since ϕ_0 takes value 1 around the point p_0 and 0 far from it, the assemblage coincides with $\tilde{A}_{p_0}$ in a neighbourhood of the point p_0. Analogously, since ϕ_1 takes value 1 on at the point p_1, the assemblage coincides with $\tilde{A}_{p_1}$ in a neighbourhood of the point p_1. The resulting operator is then a smooth transformation of $\tilde{A}_{p_0}$ into $\tilde{A}_{p_1}$. Note that this is just one of the many possible recombinations of operators.

The operator can at this point be re-projected to an operator $\&A_{p_0,p_1}$ on the substrate space.

More generally, we can define an assemblage between two or more heterogeneous patches, if there exist $k+1$ points denoted $p_0, ..., p_k$ such that the neighbourhood of each one intersects the following one. In each intersection. the previous process is applied, and a common assemblage is defined.

An inverse of the assemblage operator is the disjunction operator: $(\&)^{-1}A$, that is able to generate two distinct operators A_{p_0} A_{p_1} starting from an integrate assemblage A. This operator is not unique, since the distinct operators can be generated in different ways.

3.4 The heterogenetic flow

The differential becoming of forms in space and time is the solution u of the equation associated to the assemblage operator A:

$$\partial_t u = \&Au.$$

The solution u will take values in a space H, which will take into account material attributes, and it is allowed to change with rules similar to the ones

described for the domain. We will also assume that $A(u)$ takes values in the same set H.

The space domain (B, F) of the solution is given a posteriori with respect to the definition of operators. If the concatenation changes, the space changes accordingly, giving rise to a morphogenesis of spaces.

The flux has values in a space H, so that in the space domain, we find formed substances with a density that changes from point to point. This heterogenetic flow appears to be a cloud of formed substances continuously changing in form, density, composition, and velocity.

3.5 Harmonic embedding and polarisation of the flow

The flow itself can generate an intrinsic reference frame without the need of any external decoding. The frame can be determined as the principal or statistically independent components of the flow.

The vectors of the reference frame will form a harmonic embedding of the process itself. If we define with $\&A(u)$ the concatenation of singular differential operators, the embedding of the heterogeneous process will be defined by all the solutions of the spectral problem:

$$\&A(u_i) = \lambda_i u_i$$

where u_i are the modes of vibration proper to concatenation, also known as eigenvectors. It is therefore, within the heterogeneous process itself, the choice of the reference system in which to represent its evolution that are produced as vibrations of the process.

The instantaneous projection of the flow into its harmonic embedding is a point and its evolution is a trajectory in space that are produced as vibrations of the process.

In his book on Francis Bacon (G. Deleuze , 2003), Deleuze writes that sensibility is vibration and sensation stems from the reception of these vibrations:

"Sensation is vibration. We know that the egg reveals just this state of the body 'before' organic representation: axes and vectors„ gradients, zones, cinematic movements and dynamic tendencies, in relation to which forms are contingent and accessory".

Furthermore:

"This is why we treat the Body without organ (the heterogenetic flux) as the full egg before the extension of the organism and the organisation of the organs, before the formation of the strata". ‘ Commenting on this passage, Franco Berardi (F. Berardi , 2015) writes:

"Like a thin film recording and decifering non-verbal impressions, sensibility allows human beings to join together ... and regress to a non-specified and non-codified state of bodies without organs that pulsate in unison". "Sensibility is the faculty to decoding intensity, which by definition means to escape the extensive dimension of verbal language. Sensibility is the ability to understand the unspoken".

Interpreting literally the idea of flow vibration Deleuze suggests, sensible perception will be therefore the projection of the heterogenetic flow on the mode of vibrations of the flow itself, i.e., on its harmonic embedding. Harmonic embedding is the intrinsically decoding intensities of the flow without external decoding structures. This process does not correspond to a categorisation but to a detection of the main orientations of the flow even in absence of any stabilisation in fixed forms.

A. Sarti, G. Citti (2015) have shown that such a harmonic approach can individuate perceptual forms from visual stimuli. Extending this approach, in A. Sarti, G. Citti, D. Piotrowski, in preparation , visual plastic formants have been individuated, showing that the principal axes determine the reference system of the space in which visual semiotic will develop later.

Hence, heterogeneous flow eigenvectors have a dual status. They are intrinsic reference flow axes and are continuously varying forms, pre-patterning of a possible successive stratification.

We will analyse in the next chapter how the heterogenetic flow and its polarization along principal eigenvectors will give rise to a multiple stratification of expression/content layers.

4 Genesis of the semiotic function

We will show in this chapter how heterogentic flow and its polarization in principal axis are at the base of the constitution of the semiotic function. These axis of cohesion will construct the expression/content planes in the sense of a generalized semiotics, not necessarily related to semio-linguistic contexts. To this end, we will introduce the concept of semiotic function (section 4.1), and we will analyse how it has been studied with different approaches. We will outline the limits of such approaches to clarify the important role of heterogenesis in the constitution of the semiotic function.

4.1 The semiotic function: the classical approaches

The semiotic function, commonly defined after Hjelmslev [1] as a relation of interdependence between the planes of expression and content, and constituting the essence of semiolinguistic phenomena, remains a blind point in this field.

Even if some of the operative modalities of the semiotic function have been clearly recognized (for instance, the commutation operation or its role in the infinite dynamical semiosis—see below), its complete and deep comprehension has not yet been achieved. In fact, even when it is explicitly taken into account in a theoretical framework, the semiotic function is often reduced to operational schemas that imply it without determining it. In this way, its essential parts are relegated in a background that is not completely theorized, and finally, it is excluded from the scope of an explicit semiolinguistic knowledge. [2]

To illustrate such difficulties to the very theorization of the semiotic function and to propose a possible way to overcome them, we will quickly examine some theoretical apparatus that are historically important or currently prevalent.

The clearest illustration of the difficulties that prevent clear conceptualization of the semiotic function is provided by Benveniste, who faces the question of what it basically consists of only with metaphors: "there is such a close symbiosis between them [the concept and the sound image] that the concept [...] is like the soul of the sound image" (E. Benveniste , 1971, p. 45) or "The signifier and the signified [...] together make up the ensemble as the embodier and the embodiment [...]. This consubstantiality of the signifier and the signified [etc.]" (ibid. p. 45). Elsewhere, always referring to the relationship between expression and content, we find "fusion", "reciprocal assimilation", and "incorporation", sometimes reinforced by epithets of reciprocity (mutual, coextensive).

[1] "the [semiotic] function is in itself a solidarity. Expression and content are solidary—they necessarily presuppose each other". (L. Hjelmslev , 1969, p. 48)

[2] Commonly, semiolinguistic theories only record and exploit the correlations between forms and meanings, thus without approaching their internal principles. The undivided unit of the sign is thus generally related to a coupling, either logic or dynamics, of one signifier and one signified that are constituted independently one from the other. Thus, for instance, in construction grammars (W. Croft, D. A. and Cruse , 2004) (further to cognitive grammars of Langacker (R. W. Langacker , 2008)), the integrated unit of the word form and of its meaning results from an associating and storage process ("entrenchment") of a routine of co-actualization (of form and meaning) based on the reiteration of co-occurrences, and not from an "interpenetration" (of form and meaning). Word form and meaning are thus elaborated beforehand and independently one from the other.

4.1.1 Saussure

Saussure's approach is more instructive: if at the beginning of the Course he suggests a kind of merging (blending) of the signifier and the signified, he will quickly abandon this conception in favour of a functional architecture in which the unity of the signifier and the signified is partially rebuilt.

For him, indeed, "to consider a term simply as the union of a certain sound with a certain concept is grossly misleading" (F. de Saussure , 1959, p. 113).

And if "Neither are thoughts given material form nor are sounds transformed into mental entities" (ibid. p. 112), it is because the sign is nothing but an "edge effect". As shown in the famous schema (ibid. p. 112), the sign is the functional consequence of a superior systemic reason (the language) that creates dual units by correlating relations of reciprocal delimitation established respectively in the substances of expression and of content. We know that to overcome the conception of sign as a fusion of sound and of meaning, Saussure has developed a theory of value, but we know it also is without convincing results. In the end, the originary awareness of the unity of content and expression, which is at the foundation of semiolinguistic intuition, will have been dissolved in a theoretical device accounting for the systemic organization of a system of signs. This unity of content and expression, which is a phenomenological feature and is essential to the recognition of the semiolinguistic fact, is then relegated in the background.

4.1.2 Hjelmslev

In Hjelmslev's glossematic theory, the process is quite similar but with the merit of clarity. In fact, if the undivided unity of expression and content is again placed in a theoretical background, the process is now explicitly theorized: in the glossematic apparatus, the set of relationships on which the semiolinguistic objectivity is built is located at an analysis level that is hierarchically below the level of the connection between the planes of expression and content. In this way, the semiotic function is located outside the scope of linguistic knowledge and appears to be indeterminable. Correlatively, the relations between expression and content units (for instance, between signifier and signified) are rebuilt on the ground of the commutation's relationship, which is precisely defined as a "conjunction" between "disjunctions" recorded in each plane. One observes that once again, the primacy given to functional architecture is done at the expense of the semiotic function, yet recognized by Hjelmslev as the condition of any genuine semiotic phenomenon. [3]

[3]"An expression is expression only by virtue of being an expression of a content, and a content is content only by virtue of being a content of an expression" (L. Hjelmslev , 1969, p.

4.1.3 Husserl

It is the same for the Husserlian phenomenological analysis: the problem of the undivided unity of the sign does not find an internal answer, and it is *in fine* through a superstructure of awareness attentional field that the signifier and the signified recover a certain unit. Let us recall indeed that if, as is asserted in the first Logical Investigation, "[...] the essence of an expression lies solely in its meaning" (E. Husserl , 1995, p. 199), the phenomenological analysis is unable to recognize, inside the "sign-phenomenon", the presence, nevertheless indubitable, of a simply sensorial component, jointly given to an intentional aim of meaning (the signified). [4].

To overcome this difficulty, which consists of keeping and combining two intentional aims (one related to perception and the other one to meaning), that belong to distinct levels of awareness, Husserl takes advantage of the external structure of an attentional field in which the two aims are located at different but interdependent levels. [5]

But if in doing so, the phenomenological description of signifier and signified is partly achieved, the unity of the semiotic function is lost, simply because the two aims are pursued independently, thus contravening the type of unit enacted by the semiotic function.

4.2 The semiotic function: from Peirce to Merleau-Ponty

These three approaches have been used jointly to recognize at their starting point the essential character of the semiotic function. But over their developments, which aim at the functional determination of the units' signs, in their composition and their connections to the other units, the primary fact that semiotic function reports is misplaced or exceeded. When clarifying the functional regulations and the relational modalities that determine the phenomena of semiotics in their empirical objectivity, the first condition of any semioticity, the essence of the semiotic fact, to resume Husserl, is placed in the background as an implicit foundation. It is undoubtedly unknowable, but always contiguous to the determinations that the theoretical devices deliver. We then understand

48)

[4]"[The] consciousness of the word's sound is manifestly not the consciousness of the word. [Nevertheless] In the apprehension of the word, the first is contained; the sensible sound of the word indeed appears; but only as a founding basis." (E. Husserl , 1995, Appendice II, p. 175, our translation)

[5]In the §4a, called "les fonctions de l'attention: le remarquer primaire et le viser thématique", (E. Husserl , 1995, p. 45) we can read: "The function of the consciousness of the word's sound is manifestly not to retain in the primary mode of noticing which is accomplished within it, but to drive it towards a consciousness of meaning [i.e. thematic aim]"

why many semiolinguistic approaches have been developed by treating the semiotic function on an oblique or side mode. Such is particularly the case of the Peircean semiotic, which we will examine schematically.

4.2.1 Peirce

As is well-known, the core of the Peircean apparatus articulates three terms, of which the last (the Object) is split, namely (i) the sign (or representamen), (ii) the interpretant (which is also a sign), and (iii) the Object, in which one will distinguish two aspects: (iii-a) one pertaining to the real world and called the dynamic (or dynamoïd) object, and the second (iii-b) pertaining to the semiotic system: the immediate object. [6]

It is well known that the sign, taken as representamen, refers to the object it represents through the mediation of other signs that act as interpretants, which are then provoked by the representamen. In such a way, "[the interpretant is] a sign which returns a representamen to its object" (G. Deledalle , 1979, p. 21-22, our translation). In this sense, "[the interpretant] operates the mediation between the representamen (first) and the object (second)" (N. Everaert , 1990, p. 40, our translation).

The interpretant is thus the active principle of the semiosis in that it establishes the link between the representamen and what the representamen refers to. We know that this functional configuration opens an unlimited semiosis process: the interpretant, as a sign, calls to others interpretants, and so on, endlessly.

We now examine the relationship between signifier and signified in this theoretical device. In the Peircean apparatus, the role of signifier is clearly taken by the representamen, which is "the sign as it is *presented* and that the interpretant will refer to the object it represents" (G. Deledalle , 1979, p. 23, our translation).

Concerning the role of the signified, the case is more complex because the Peircean device is dynamic, and the content assigned to a sign is the asymptotic limit of an endless semiosis process. Depending on whether one is interested in a state of the signified corresponding to a given stage of the semiosis process or corresponding to the limit of the infinite semiosis, the role of the signified will be carried respectively by the interpretant or by the immediate object: "It seems natural to use the word meaning [signified] to denote the intended interpretant of a symbol" (C. S. Peirce , 1931-1935, p. 31-35), and elsewhere, "the complete immediate object is identified with the signified" (U. Eco , 1980, p. 108, our translation).

[6] "It is usual and proper to distinguish two Objects of a Sign, the Mediate without, and the Immediate within the Sign" (C. S. Peirce , 1931-1935, p. 406).

Whatever the option is, the one reporting the signified to the interpretants or to the dynamic object (i.e., the object that a series of interpretants gradually circumscribed), the semiotic principle is carried by the representamen because it, in its quality of sign, opens to other signs, or, according to the canonical definition of the sign, “determines” interpretants that contribute to configuring an immediate object.

In this context, the semiotic function is moved on a phenomenological plane: the representamen essentially implies an opening towards another than itself, sign or immediate object, and this characteristic is expressed in the very moment of its donation as a sign because it configures its appearing. This is why we must recognize with Eco that "the term ’meaning’ seems at once a semantic category and a category of the phenomenology of perception" : it is only because I know that smoke means fire "[that I] am able to render the sensory datum meaningful, by seeing it as that smoke which can reveal fire" (U. Eco , 1984, p. 33).

To conclude temporarily: we see that, similarly to the theoretical frameworks previously discussed, the Peircean conception on the one hand accounts for the (dynamical) organisation of meaning and signs and on the other hand sets the semiotic function on a phenomenological plan, which escapes the determinations of the theoretical framework. But it will be useful not to stop the discussion on the Peircean device here and to extend it by examining the interpretation that, in the light of the Peircean design, Eco proposes of the Hjelmslevian apparatus.

4.2.2 Eco/Hjelmslev

This time, the question is no more about signs, but about the triad form, substance and matter (or purport). What is at stake here is the constitution of substances and, in the end, the possibility of understanding the internal principle of the semiotic function.

Let us first recall the three notions in the Hjelmslevian apparatus. The form is an ideal structure, specifically an abstract network of dependencies. When this form becomes incarnated and manifested, it is precisely denoted by the concept of substance. The third term, the purport, is related to the amorphous manifold that is modelled by the form when it is projected on it to produce the substance.

In glossematic theory, the purport is defined as an amorphous aggregate of independent and unitary atoms. In defining purport in such a way, Hjelmslev locates it at the boundaries of what can be known. On the one hand, as Hjelmslev outlines [7] , knowledge concerns only “cohesive” relationships that do not belong to purport’s units. On the other hand, the purport can be

[7] "for the aim of science is always to register cohesions, and if an object only presents the

conceptualized since, because it is apt to receive forms, it must hold the qualities by which such an instantiation is possible. Thus, even if it is free of form, the purport is minimally formed (as a set of univocal and mutually untied atoms) to constitute the homogeneous soil for possible actualization of forms.

What Eco reconsiders is the principle of two distinct purports, the one of expression and the other of content, that would be respectively the receptacles of the expression and content forms to produce the corresponding substances of expression and content. From Eco's point of view, what Hjelmslev calls purport corresponds to the Peircean dynamical object, simply because, like the dynamical object, the purport in the Hjelmslevian conception escapes all knowledge and constitutes a field to be "semiotized". Indeed, Peirce defines reality as "[...] the limit of what can be known, what would be known by an infinite semiotic practice" (N. Everaert , 1990, p. 45) and considers the dynamical object as "[...] what the sign refers to in its existential singularity" (G. Deledalle , 1979, p. 66).

The Hjelmslevian purport is similarly a manifold of singularities without any form or cohesion, and then it is located outside the field of any knowledge. Conceived in this way, the purports of expression and content cannot be distinguished one from another, since they are defined in the same way: they are amorphous aggregates, as untied punctual diversities, and do not hold any organizational characteristics that discriminate them.

Starting from there, we will follow Eco's thesis that "represents the continuum of the expression and the continuum of the contents as a same entity" (U. Eco , 1988, p. 80, our translation) : "The matter, the continuum about which and through which signs speak, is always the same. It is the Dynamic Object that Peirce talked about [...]" (U. Eco , 1988, p. 44). Consequently, "the continuum which one forms to express itself is the same one than that which one expresses" (U. Eco , 1988, p. 80). So be it.

At this point, we note that this theoretical adjustment, introducing the assimilation of the purport of expression and contents, is insufficient to enlighten the semiotic function, since this function is committed between the planes of expression and of contents through their articulations of substance/forms, and does not imply in any way, other than in an atheoric background (see above), the presence of the purport. Furthermore, this is clearly represented in Eco's diagrams: it is within the interior disc, subdivided into two half discs, one for expression, the other for contents, that are constituted the units of form and substance, respectively, of expression and contents, and that their

possibility of registering constellations or absences of function, exact treatment is no longer possible" (L. Hjelmslev , 1969, p. 83).

connection (the semiotic function) is performed. The part between the exterior and interior circles, which thus represents the common purport, is not implied in the elaboration of the links between units of the expression and contents planes, at least directly and formally. And it is on this latter point that the theoretical reconfiguration that Eco operates is essential.

4.2.3 Eco/Peirce

What is at stake now is the relationship between the dynamic object and the immediate object, i.e., the relationship of reality to what one expresses of it, or in other words, using Hjelmslevians terms, the relationship between the purport and the form.

Concerning this point, on the side of glossematic, the question is clear: the purport constitutes a completely passive receptacle and is able to receive any semiotic formation. That is to say, the matter does not express by itself. The Peircean point of view is quite different. As we have seen, "the immediate object is the mode of donation [i.e., the meaning as defined by Frege] of the dynamic object" (U. Eco , 1988, p. 108, our translation).

But this mode of donation, which is a certain point of view of the object, is not, according to Peirce, arbitrary: it is not decided in the sign system but emanates from the dynamic object itself: "It is the dynamic object which determines the representamen to represent it through a certain point of view, the one of the immediate object" (N. Everaert , 1990, p. 44). Then there is a first experience of the world (dynamic object) that originarily meets a universe of tensions, balances and constraints, waiting to be constituted as qualified phenomena, but that already orients "a certain point of view" of it.

Thus, one can say that "[. . .] it is under the pressure of the world (as a dynamic object) that the sign represents the world [. . .] [8] and that "[...] the immediate object gives account of an already implicit meaning inside the dynamic object" (U. Eco , 1988, p. 108, our translation). It will then be necessary to question the meaning and the statute of this "implicit", which is, according to Merleau-Pontian's terms, like a "preparation to the object".

This Peircean conception becomes even stronger when regarding the difficulties encountered by the converse positions developed in the Hjelmslevian apparatus (see further). From Peirce we will retain the conception of a first ensemble of solicitations, a first fabric of dubious impressions, an expectation of reactions and positioning, that gradually, in the way of individual experiences, take form, meaning and even a statute of object.

[8](N. Everaert , 1990, p. 44), and Eco: "les signes sont produits sous la pression de l'expérience du monde (comme objet dynamique)" (U. Eco , 1980, p. 75).

Moreover, this option finds other supports when examining certain contemporary approaches to the semiotic function, notably the thesis of Fontanille.

4.2.4 Fontanille

From Fontanille's point of view [9], in agreement with recent developments in semiotics, semiosis is basically a matter of body. In fact, after he has observed that "the body explicitly came back in semiotics" (J. Fontanille , 2006, p. 12), he continues that "the anchoring of semiosis (is) in the sensible experience" (ibid.). More precisely: "as soon as we wonder about the operation which joins together the two planes of a language, the body becomes essential[:] it [the body] has to be considered as the only instance common to the two faces [signifier/signified] or to the two plans [expression/content], and which can ground, guarantee and carry out their union in a meaning unit" (J. Fontanille , 2004, p. 13).

The body is thus conceived as an "operator of semiosis" in different ways. First, the body takes part in the elaboration of sensible qualities of which it constitutes the praxical side: "each sensory apprehension is an apprehension of the motion, which accompanies, precedes or causes the motion, and which, consequently, is originarily a sensation of the flesh and of the body motion" (J. Fontanille , 1999, p. 9). We see that at this first level of correlation between the sensory experience and body commitments, sensible qualities, are intrinsically meaningful, being praxical values.

But it is only in subsequent operations (of conversion) that significances attached to axiologic dimensions (note that axiology generally means theory or description of systems of values (ethical, logical, esthetical and more generally anthropological)) are processed and assigned. What we observe on this second level and at the subsequent ones is that semiosis is conceived as a process that, by means of the power of the body and its affects, processes and "computes" new values of content to a plan of expression previously made up. In this perspective, there is no longer a semiotic function as we have introduced, but a process of semiotization through a reconfiguration and attribution [10] of values

[9] Fontanille never misses an opportunity to underline the central role of the semiotic function: "let us take care of the fact that [analysis] respects the minimal constraint of a solidarity between expressions and contents" (J. Fontanille , 2006, p. 14)

[10] "[conversions] are operations which imply an epistemological subject equipped with a body, which perceives significant contents and which calculates and projects their values. For each change of level of pertinence, one can attribute the re-articulation of meanings to the activity of this sensitive and "embodied" operator: he perceives the meanings of a first level as tensions between categories, as graduated conflicts, and he draws from this perception new meanings, articulated as "positional values", on the next level of pertinence". (J. Fontanille , 2004, p. 14).

of meaning. In Fontanille's view, we have to place ourselves at the stage in which the body motions are accomplished correlatively with the installation of meaningful sensible qualities to see at work a genuine semiosis, which lacks at the level of conversion (second and further levels).

We are not interested here in discussing the relevance of such an approach, since its interest and its efficiency have to be established elsewhere. We will just observe that the semiotic operation that is at its base and constitutes the first layer of expressivity remains obscure. Especially, we will observe that, set in that way, the problem of the semiotic function has been faced in the problematical frame of the "first" Merleau-Ponty problematical frame towards which we will quite naturally be redirected.

4.2.5 Merleau-Ponty

Against empiricism [11] and against intellectualism [12] according to which the meaning of the perceived world does not reside within it, Merleau-Ponty defends an expressivist position, arguing that the sensible qualities relate to the vital significations that constitute the originarily framework of experience: "[In experience] we are not given 'dead' qualities, but rather active properties" (M. Merleau-Ponty , 2012, p. 52), and in this "layer of living experience through which other people and things are first given to us" (ibid., p. 57), it is qualities inhabited by an existential value, by a "meaning for us" that "sensing" apprehends (ibid., p. 52).

What is encountered in an immediate manner is therefore not a mosaic of mute sensations to be explored or informed, but indeed a fully signifying presence: "[the roots of perception] do not consist in the 'elements' of sensuous impression, but in originary and immediate expressive characters. Concrete perception [...] is never resolved into a simple complex of sensuous qualities [...] but each time accords itself with a determined and specific tonality of expression". (Cassirer in (Rosenthal, Visetti , 2008, p. 185))

To elucidate the expressive fact, to ground in law and in reason the fact of a tangible presence of meaning, we know that the Merleau-Pontian solution consists of placing oneself before the moment when sensible qualities are constituted as signifying, that is, this originary moment of a face-to-face between the life force of "one's own body" and an environment of uncertain

[11] According to which the world is originarily delivered as a manifold of sensations, in which the perceptive faculty spontaneously distinguishes contiguities, regularities, and resemblances, and, by means of associations, produces in the mind a world of things.

[12] According to which the immediate matter of sensations is likewise a mosaic of qualities, but in which thought installs a universe of determinate objects by means of constitutive syntheses governed by sovereign concepts.

solicitations, of "poorly formulated question[s]" (M. Merleau-Ponty , 2012, p. 242) to which one's own body attempts to respond in search of syntony: "Thus, a sensible that is about to be sensed poses to my body a sort of confused problem. I must find the attitude that will provide it with the means to become [some] determinate [quality]; I must find the response to a poorly formulated question. And yet, I only do this in response to its solicitation. My attitude is never sufficient to make me truly see blue or truly touch a hard surface. The sensible gives back to me what I had lent to it, but I received it from the sensible in the first place" (ibid.).

Thus, "all begins" with an interested and interrogative meeting between a bodily schema and an environment of solicitations, a surrounding halo of singular tensions (which, as we will see, directly relates to Deuleuzian conceptions), one that directs towards a constitution made of the crossings of body and world, and having, from the onset, a value as co-expression.

It is therefore necessary to look prior to one's relationship as an instituted individual with one's surroundings as a defined set of objects and of qualities. And prior to such a relationship, there remains but one's body as a carrier and performer of a certain life force and a hazy environment that "vaguely solicits" a sort of "poorly formulated question" (ibid., p. 222). "Without the exploration of my gaze or my hand, and prior to my body synchronizing with it, the sensible is nothing but a vague solicitation" (ibid.) with which I will attempt to syntonize and the effect of which will flourish into intrinsically significant sensible qualities.

Thus, a world is constituted as a "background", that is, as a world in which the forms and qualities signify the acts and engagements of the subject who installed them as such as a sort of successful "coupling" of one's own body with its pre-sensible and pre-objectal environment, forms and qualities that, in turn, draw something of a geometry (the "background"), the forces of the lines of which express the possible involvements and actions of the subject.

4.3 The heterogenetic hypothesis

We hope that the overview we carried out of some of the major semiolinguistic theories, even if limited and schematic, can help identify some of the main obstructions to understanding the constitution process of the semiotic function. We hope it can suggest some tracks to overcome them.

Thus, from the examination of glossematics, we will retain the deadlock induced by the reduction of purports to an aggregate of undifferentiated and homogeneous atomic units, in which, just by a set of relationships, the purport would be informed to produce substances). From the analysis of the Peircean apparatus, we will retain the importance to conceive an originary unspecified

purport that exercises a "pressure" to constitute itself in its meaning and its phenomenal forms.

Finally, the approach of Fontanille, together with a general movement in semiolinguistic science, identifies the body as a "strange signifying machine" (M. Merleau-Ponty , 2012, p. 114) accordingly to Merleau-Ponty's terms.

To deploy both this approach and the Peircean intuition of a dynamical object as experiential unspecified background "which questions the body", we will turn to Merleau-Ponty, who has produced some of the deepest reflections about meaningful morphologies and, more generally, about the semiotization of the world in relationship with a body. This will constitute our background to conceive the emergence of the semiotic function in terms of an heterogenesis.

4.3.1 Towards heterogenesis

For this purpose, let us reconsider the Hjelmslevian device. As we have seen, it locates the semiotic function "above" forms and substances, therefore out of the field of semiolinguistic knowledge. If Hjelmslev proceeds in this way, it is for good reason.

In fact, the semiotic function is not a phenomenon in the sense of empirical knowledge: the semiotic function does not let itself apprehend like a substance of which the form has to be identified or of which the functioning laws have to be discovered: its nature is of another kind. At the same time, one of the main obstacles with which semiotic knowledge is confronted comes from the fact that it considers the planes of expression and contents already equipped with its own forms and substances. With such gnoseological *a priori*, their unity becomes unthinkable. It is thus before any concept of form and purport and before any structure of empirical rationality that is correlative to the *a priori* concepts of form and purport that the foundation of the semiotic function must be sought.

This is the problematic line that attracts us to Merleau-Ponty, who, as we have seen, considers the co-occurrent constitution of a body and a world, both resulting from a set of interactions. The body, initially set as "muffled" vital power, and answering to uncertain requests of a surrounding that questions it, informs this surrounding of its own rhythms, of its specific conduits. It is a world of sensorial qualities that is then established. In this process of co-constitution, sensorial qualities are, by construction, intrinsically meaningful: the sensible is from the beginning equipped with a signification that is assigned to him by the body matrix that brings it into existence. And the world in its native fabric is a world of expressions, i.e., a world already endowed by value and meaning.

To progress in this direction, it will thus be advisable to pose the question at a problematic level placed before the Hjelmslevian concept of "undifferentiated" purport.

Indeed, let us recall that, defined as a "constellation" of univocal entities, matter configures itself as "homogeneous" in the sense that all its elements share a common nature and common qualities. These undifferentiated elements are then linked in a unitary and coherent mass, an "amorphous" mass that is ready to receive a form.

Heterogenesis challenges both the idea of an in-shaping form and the *a priori* of homogeneity and continuity of the purport.

Finally, it means to choose an *a priori* of heterogeneity, which precedes the interplay between forms and purports. In other words, it means to be positioned on a theoretical plane in which still there is the possibility to produce substances. Notice that this is in narrow resonance with the position of Merleau-Ponty, for which there is a myriad of mutually irreducible, singular and unqualified solicitations that are originarily offered to our vital behaviours. It is then impossible to conceive them in the form of simple undifferentiated units, which would give rise to a homogeneous purport. Thus, whether it is the case of the originary installation of a signifying world in relationship to body (Merleau-Ponty) or the primordial fact of an interpenetration of the planes of expression and of content (Hjelmslev-Deleuze), it is always before all constituents or constituted dimensions that we must look for the emergence of the semiotic function, particularly before the setting of univocal and explicit formal regimes, before the hypothesis of first units making "homogeneous" matter (Hjelmslev), and before stable and determined sensible qualities (Merleau-Ponty).

The examination of the semiotic function must then be expected at a level in which a multitude of local tensions, mutually irreducible (in the sense that they do not create a common material), constitutes the primordial environment, and in which, by a sort of progressive tightening towards homogeneity and continuity, the constitution of flows or aggregates can be envisaged.

4.3.2 From assemblages to substances: a formal morphodynamical process

This idea of emergence of different morphodynamical lines from the actualisation of assemblages of heterogeneous operators corresponds exactly to Deleuze's position about the constitution of the espression/content planes:

"The diagrammatic multiplicity can be realized and the differential of forces integrated only by taking *diverging* paths, splitting into dualisms, and following lines of actualisation without which everything would remain the dispersion of an unrealized cause. ... It is precisely because the immanent cause, in both its

matter and its functions, disregards form that it is realized on the basis of a central differentiation which, on the one hand, will form visible matter, and on the other will formalize articulable functions". (G. Deleuze , 2006) [13]

It is then the diagrammatic multiplicity, i.e., the heterogeneous assemblage that has to be actualised in a flux presenting diverging paths constituting expression/content planes.

Deleuze challenges the concept of a proper body of phenomenological tradition towards a new idea of body that is open to a constant reorganisation (the Body without Organ (BwO)). This reorganisation is due to the presence of an "outside" of organic and inorganic forces and corresponds to the possibility of recombination of the assemblage of heterogeneous operators. The BwO is a body open to desire, in which to desire means to make assemblages. Then the BwO is an assemblage open to recombination. Deleuze stresses the possibility of inventing a body just by recombining heterogeneous virtual elements and at the same time converges towards Merleau-Ponty's idea that through the assemblage/body occurs an emergent co-constitution of the sensible and the articulable. By virtue of mathematical instruments introduced in Section 3, we can express the genesis of the semiotic function as a true formal *morphodynamics of heterogeneous assemblages with a divergent actualisation.*

In fact:

- Operators are locally defined in space and time in Section 3.1, allowing heterogeneity both in space and time. Heterogeneity has been considered both from the point of view of geometry of space phase and from the point of view of dynamics.

- A formal definition of assemblage $\&A(u)$ as a composition of heterogeneous operators has been given in Section 3.3. This definition has been possible thanks to the theorem of Rotschield and Stein, in which the composition of operators with different internal geometry is produced by means of lifting of vector fields (see Section 3.2).

The multitude of differential operators, mutually irreducible, that are defined locally and concentrate on, in their intensive sense, universes of possible forms, constitute the main part of the "miscellaneous native of local tensions " previously envisaged.

- This construction of the heterogeneous assemblage can be actualized (or integrated) to give rise to a flux u, continuous becoming of forms in space and time, as stated in Section 3.4.

[13]Note that in the English translation of Sean Hand, the French words "differentiation" and "differenciation" have been translated both with the English term "differerentiation", losing the fundamental difference between the two terms, since the French word "differenciation" has to be translated as "actualization" or "integration", as Deleuze explains (G. Deleuze , 1994, p. 208-214). We followed the correct meaning suggested by Deleuze.

This fact proves that the space between (symmetric, homogenous) structures and chaos is not void. There is always the possibility to create forms by actualizing assemblages of non-homogenous, non-symmetric operators that are locally defined. The flux u has all the characteristic of a morphological field with an internal consistency, since it is the integration of a differential problem. Then, it tends to create coherent forms, but these are continuously changing and are never stabilized in true gestalten. The flux is at the base of a complex theory of becoming that Deleuze invites us to practice at a social, psychic, neurophysical, artistic, and mathematical level.

- Two time axes are present, one to actualise and the other to recombine differential constraints. This allows a continuous recomposition of the assemblage and the introduction of the dimension of externality ("outside" of organic and inorganic forces).

- The process of divergent actualisation to form expression/content planes during the integration of the assemblage is engendered by the emergence of two or more principal components in the actualisation of the heterogenetic flux.

Principle components testify for internal dimensions of aggregation and cohesion of the flux. Principal modes of vibrations of the flow $\&A(u_i)$ introduced in Section 3.5 now supply the main orientations of coherence of the flow that will progressively constitute its substances. These orientation vectors are the generators of the embedding, the intrinsic system of reference of the heterogeneous morphodynamics.

If we consider just the two principal dimensions (or a combination of vectors to be grouped in two principal dimensions), they could represent the polarisation in two substances envisaged by René Thom in terms of saliences and pregnances or by Merleau-Ponty in terms of the body and the world. But since the basis of the embedding is constituted by a multiplicity of vectors, they can also implement the more complex stratification of E/C substances foreseen by Hjelmslev/Deleuze. Also in this case. there is not a univocal choice of the vectors, and any grouping of them can give rise to a different stratification.

In such a way, the embedding is a protosemiotic space already carrying a sense, since it is polarised through the principal vectors E/C, which does not imply a symbolic level, but it prepares to it.

The embedding with its principal vectors is now the manifold in which the constitution of sign can take place, eventually through the installation of oppositive morphodynamical systems, as in the tradition of structural morphodynamics. Structural dynamical systems will act on the heterogenetic flow as a dispositive of control, aiming to stabilize the flux in an *a priori* defined structure of attractors.

The problem of the installation of such a categorisation device is outside the scope of the present work, and it has been examined recently, for example,

in (D. Piotrowski , 2017).

This question as well as all the other topics presented in this study will be examined in extended form in a forthcoming publication (A. Sarti, G. Citti, D. Piotrowski, in preparation)

References

F. Bailly, G. Longo, The physical singularity of life, Imperial College Press, 2008.

J.Bennett, Vibrant matter, Duke University Press, 2010

E. Benveniste, Problems in general linguistics (M.E. Meek, Trans.). Coral Gables: University of Miami Press, 1971.

F. Berardi, And, phenomenology of the end, MIT press, 2015.

J.-M. Bony, Principe du maximum, inégalité de Harnack et unicité du problème de Cauchy pour les opérateurs elliptiques dégénérés, Ann. Inst. Fourier 19, 227–304, 1969.

R. Braidotti, Metamorphoses: Towards a Materialist Theory of Becoming, Cambridge: Polity Press; Malden, MA: Blackwell Publishers, 2002.

L. Capogna, G. Citti, Regularity for subelliptic PDE through uniform estimates in multi-scale geometries. Bull. Math. Sci. 6, no. 2, 173–230, 2016.

G. Citti, A. Sarti, A cortical based model of perceptual completion in the roto-translation space. J. Math. Imag. Vis. 24(3), 307-326, 2006.

G. Citti, A. Sarti, Neuromathematics of Vision, Springer Publisher, 2014.

G. Citti, C^∞ regularity of solutions of a quasilinear equation related to the Levi operator. Ann. Scuola Norm. Sup.Pisa Cl. Sci. (4) 23, no. 3, 483–529, 1996.

G. Citti, E. Lanconelli, A.Montanari, Smoothness of Lipchitz-continuous graphs with nonvanishing Levi curvature. Acta Math. 188, no. 1, 87–128, 2002.

G. Citti, M, Manfredini, Blow-up in non homogeneous Lie groups and rectifiability. Houston J. Math. 31, no. 2, 333–353, 2005.

W. Croft, D. A. and Cruse, Cognitive Linguistics. Cambridge: Cambridge University Press, 2004.

G. Deledalle, Théorie et pratique du signe, Introduction à la sémiotique de Ch. S. Peirce, Paris : Payot, 1979.

G. Deleuze, Difference and Repetition, Columbia University Press, 1994.

G. Deleuze, Empiricism and subjectivity, Columbia University Press, 2001.

Foucault translated and edited by Sean Hand University of Minnesota press, 2006.

G.Deleuze, The Logic of Sensation, London Continuum, 2003.

G. Deleuze, F. Guattari, A Thousand Plateaus, University of Minnesota Press, 1987.

G. Deleuze, F. Guattari, What is philosophy?, Verso, 1994.

R. Duits, E.M. Franken: Left invariant parabolic evolution equations on SE(2) and contour enhancement via invertible orientation scores, part I: Linear left-invariant diffusion equations on SE(2). Q. Appl. Math. 68, pp. 255-292, 2010.

U. Eco, Peirce et la sémantique contemporaine, Langages, n°58, 1980.

U. Eco, Semiotics and Philosophy of Language, London: MacMillan Press, 1984.

U. Eco, Sémiotique et philosophie du langage, Paris: PUF, 1988.

N. Everaert, Le processus interprétatif, Introduction à la sémiotique de Ch. S. Peirce, Paris : Mardaga, 1990.

Fabbri, P., L'oscuro principe spinozista: Deleuze, Hjelmslev, Bacon, Da: Discipline Filosofiche, numero dedicato a G. Deleuze, n. 1, 1998.

J. Fontanille, Modes du sensible et syntaxe figurative, Actes Sémiotiques, n°61-62-63, 1999.

J. Fontanille, Soma et séma, Figures du corps, Paris : Maisonneuve et Larose, 2004.

J. Fontanille, Pratiques sémiotiques: immanence et pertinence, efficience et optimisation, Nouveaux Actes Sémiotiques, n° 104, 105, 106, 2006.

A. Friedman, Partial differential equations of parabolic type, Prentice-Hall, Inc., Englewood Cliffs, N.J., 1964.

D. Gilbarg, N. S. Trudinger, Elliptic partial differential equations of second order. Reprint of the 1998 edition.

R. Godel, Les sources manuscrites du Course de Linguistique Generale de F de Saussure, Geneve:Droz, coll. Publications Romanes et Francaises 61, 1969.

L. Hjelmslev, Prolegomena to a theory of language (F. J. Whitfield, Trans), Madison: University of Wisconsin Press, 1969.

W.C. Hoffman, The visual cortex is a contact bundle. Appl. Math. Comput. 32, pp.137-167, 1989.

L. Hörmander, Hypoelliptic second order differential equations, Acta Math., n.119, pp.147-171, 1967.

E. Husserl, Logical investigations: Prolegomena, Investigations I, II. (Findlay, J. N., Trans.). London & New-York: Routledge, 2001.

E. Husserl, Lecones sur la theorie de la signification, Paris:Vrin, coll. Biblioteque de textes philosophiques, 1995.

I. Krtolica, "L'algèbre de la pensée pure ». Deleuze et le calcul des problèmes, Revista Trágica: estudos de filosofia da imanência, Vol. 8 – nº 2, 2º quadrimestre de 2015.

A. Laurent, L'individualisme méthodologique, Que sais-je ? n° 2906, Paris, PUF, 1994.

R. W. Langacker, Cognitive Grammar: A Basic Introduction. USA: Oxford University Press, 2008.

A. Longo, Le modele mathematique a la base de la philosophie de Deleuze permet-il d'acceder a une realite en soi?, implications-philosophiques.org, 2016.

G. Longo, M. Montevil, Perspective on organisms: biological time, symmetries and singularities, Springer, 2014.

T. May, "Gilles Deleuze, Difference, and Science", in Continental Philosophy of Science (ed G. Gutting), Blackwell Publishing Ltd, Malden, MA, USA, 2005.

M. Merleau-Ponty, Phenomenology of perception (D. A. Landes, Trans.), London: Routledge, 2012.

A. Nagel, E. M. Stein, S. Wainger, Balls and metrics defined by vector fields. I. Basic properties, Acta Math., n.155, pp. 103-147, 1985.

C. S. Peirce, The Collected Papers, Electronic Edition (available on line), 1931-1935.

J. Petitot, Complex methodological individualism , Cosmos-Taxis, 2015.

J. Petitot, The formalisation of semiotic elementary structures, in Dario Compagno, Quantitative semiotic analysis, Springer Publisher, 2017.

J. Petitot, Y. Tondut, Vers une neurogéométrie. Fibrations corticales, structures de contact et contours subjectifs modaux. Math. Sci. Hum. 145, pp.5-101, 1999.

J. Petitot, Morphogenesis of meaning, Series "European Semiotics" (P-A. Brandt and W. Wildgen eds), Peter Lang, Bern, Volume 3, 2004.

D. Piotrowski, Morphogenesis of the Sign, Springer Publisher, 2017.

L. Rothschild, E. M. Stein, Hypoelliptic differential operators and nilpotent groups, Acta Math., n. 137, 247-320, 1976.

V. Rosenthal, Y.-M. Visetti, Modèles et pensées de l'expression: perspectives microgénétiques. Intellectica, 50, 177-252, 2008.

A. Sarti, G. Citti, "The constitution of perceptual units in the functional architecture of V1", J.Comput. Neuroscience, 38(2), pp. 285-300, 2015.

A. Sarti, G. Citti, J. Petitot, "The Symplectic Structure of the Primary Visual Cortex", Biological Cybernetics. vol. 98, pp. 33 – 48, 2008.

A. Sarti, G. Citti, D. Piotrowski, "Differential Heterogenesis: Post-structural dynamics and singular becoming", Springer, in preparation.

A. Sarti, F. Montanari, F. Galofaro ed., "Individuation and Morphogenesis", Springer Publisher, 2015.

A. Sarti, D. Piotrowski, "Individuation amd Semiogenesis: an interplay between geometric harmonics and structural morphodynamics", in "Individuation and Morphogenesis", Springer publisher, 2015.

A. Sarti, D. Barbieri, "Neuromorphology of meaning", in "Quantitative and qualitative practices in contemporary semiotic research", ed. Dario Compagno, Springer publisher, 2017.

F. de Saussure, Course in general linguistics (W. Baskin, Trans.), New York: Philosophical Library, 1959.

A. Sauvagnargues, Deleuze, l'empirisme transcendantal, Paris, PUF, 2008.

G. Simondon, L'individu et sa genèse physico-biologique (l'individuation à la lumière des notions de forme et d'information), Paris: PUF, 1964; second ed. J.Millon, coll. Krisis, 1995.

E. M. Stein, Harmonic Analysis: Real-Variable Methods, Orthogonality, and Oscillatory Integrals. Princeton University Press, Princeton, N.J., University of Tokyo Press, Tokyo, 1993.

R. Thom, Stabilité structurelle et morphogenèse, Benjamin, New York, Ediscience, Paris, 1972.

R. Thom Modèles mathématiques de la Morphogenèse, Paris, Christian Bourgois. 1980.

A. Turing, The Chemical Basis of Morphogenesis, Collected Works, 4, 1-36, North-Holland, 1952-1992.

S.W. Zucker, Differential geometry from the Frenet point of view: boundary detection, stereo, texture and color. In: Paragios, N., Chen, Y., Faugeras, O. (eds.) Handbook of Mathematical Models in Computer Vision, pp.357-373. Springer, US 2006.